

**FOR OFFICIAL USE**

Score for accuracy	×	Mult. factor for speed	=			
<div style="border: 1px solid black; width: 50px; height: 50px; margin: 0 auto;"></div>		<div style="border: 1px solid black; width: 50px; height: 50px; margin: 0 auto;"></div>		<div style="border: 1px solid black; width: 50px; height: 50px; margin: 0 auto;"></div>	Team No.	<div style="border: 1px solid black; width: 50px; height: 50px; margin: 0 auto;"></div>
		+		Bonus Score	Time	<div style="border: 1px solid black; width: 50px; height: 50px; margin: 0 auto;"></div>
		Total Score		<div style="border: 1px solid black; width: 50px; height: 50px; margin: 0 auto;"></div>		<div style="border: 1px solid black; width: 50px; height: 50px; margin: 0 auto;"></div>
				<div style="border: 1px solid black; width: 50px; height: 50px; margin: 0 auto;"></div>	Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特别声明，答案须用数字表达，并化至最简。

1. There are 100 light bulbs labeled from 1 to 100, and there are 100 students in the class. Each student takes a turn to press the switch buttons of the light bulbs with a label that is a multiple of their assigned number. For example, the first student presses the switch buttons of the light bulb with label 1 and all of its multiples, the second student presses the switch buttons of the light bulb with label 2 and all of its multiples, and so on. Each student will only come out once, and if a light bulb is on, it becomes off after being pressed, and vice versa. All the light bulbs are off at the beginning.  $X$  is the number of light bulbs that are on after the 100th student presses. Find the value of  $X$ .

有 100 个灯泡，编号从 1 到 100。班上有 100 名学生。每个学生轮流按下灯泡开关，程序如下：第一个学生按下编号为 1 及其倍数的灯泡开关，第二个学生按下编号为 2 及其倍数的灯泡开关，以此类推。每个学生只出来一次。如果灯泡亮着，按下开关后就会熄灭，反之亦然。一开始所有灯泡都是熄灭的。 $X$  代表在第 100 个学生按下开关后，灯泡亮着的数量。求  $X$  的值。

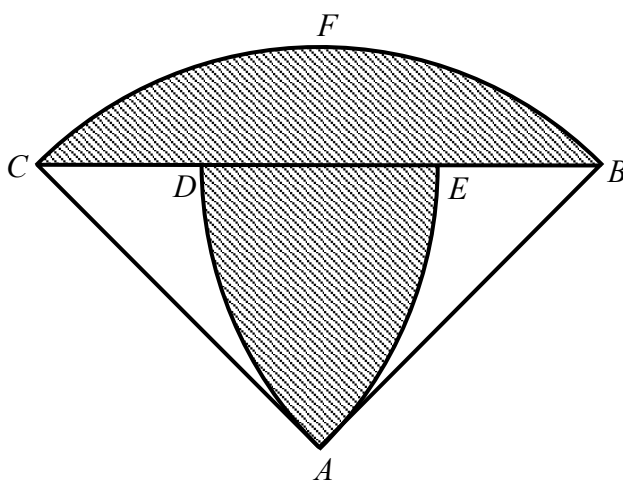
$X =$

2. Given  $x + \frac{1}{x} = 2\sqrt{5}$ . Find  $x^5 - \frac{1}{x^5}$ .

已知  $x + \frac{1}{x} = 2\sqrt{5}$ 。求  $x^5 - \frac{1}{x^5}$ 。

3. In the following figure,  $ABC$  is an isosceles right triangle with  $AB = 2$  and a right angle at  $A$ . The figure includes three arcs: arc  $BFC$ , arc  $AD$ , and arc  $AE$ . Arc  $BFE$  has a radius of  $AB$  and is drawn from centre  $A$ . Arc  $AD$  is drawn from centre  $B$  with radius  $AB$ , while arc  $AE$  is drawn from centre  $C$  with radius  $AC$ . The shaded region of the following figure, formed by the isosceles right triangle and the arcs, needs to be determined. Find the area of this shaded region. (Use  $\pi = 3$ )

下图中， $ABC$  是一个等腰直角三角形及其角  $A$  为直角， $AB = 2$ 。图中有三个弧，它们分别是弧  $BFC$ 、弧  $AD$  和弧  $AE$ 。弧  $BFE$  是以  $A$  为圆心、 $AB$  为半径画出的。弧  $AD$  是以  $B$  为圆心、 $AB$  为半径画出的。弧  $AE$  是以  $C$  为圆心、 $AC$  为半径画出的。这个等腰直角三角形和这三个弧组成了以下的图形。请计算这个图形的阴影部分面积。（请用  $\pi = 3$ ）




4. Using the sequence of positive integers 1, 2, 3, 4, 5, 6, and so on, a new integer is formed by concatenating them: 123456789101112131415161718... The leftmost digit in this integer is defined as first digit. What is the digit at position 2023?

使用正整数序列 1、2、3、4、5、6 等等，通过将它们连接起来形成一个新的整数：123456789101112131415161718... 这个整数的最左边的数位被定义为第一个数位。问在第 2023 数位是 0 至 9 的哪一个数？

**FOR OFFICIAL USE**

Score for accuracy	×	Mult. factor for speed	=			Team No.	
<div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div>				<div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div>		<div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div>	<div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div>
		+		Bonus Score		Time	
				<div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div>		<div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div>	<div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div>
		Total Score		<div style="border: 1px solid black; width: 40px; height: 40px; margin: 0 auto;"></div>		Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特别声明，答案须用数字表达，并化至最简。

1. Find the minimum value of  $x^3 + y^3$  if  $x$  and  $y$  are two positive integers whose sum is 15.  
假如  $x$  和  $y$  都是正整数且它们的和是 15，找出  $x^3 + y^3$  的最小值。

2. A cubic dice has faces marked with numbers from 6 to 11. The dice was rolled twice. At the first time, the sum of the numbers on the four lateral faces was 36. At the second time, the sum was 33. What number is on the face opposite to the one with the number 10?  
有一颗骰子，它的六个面上分别写上数字 6 至 11。现投掷这颗骰子两次，第一次得知四个侧面的数字和是 36，第二次的数字和是 33。请问数字 10 的对面是甚么数字？

3. Find the greatest common divisor of  $10^{12} + 809$  and  $10^{10} + 8$ .  
找出  $10^{12} + 809$  和  $10^{10} + 8$  的最大公因子。

4. Hong Kong is located at  $(0, 0)$  of a grid map and a typhoon is at  $(4, -2)$ . Suppose the typhoon will only move to the west (left) with a probability of 0.1 or to the north (up) with a probability of 0.9, and may only change course after moving one unit distance. What is the probability that it will hit Hong Kong? (Give your answer in 4 significant figures.)  
在直角坐标平面上，香港的坐标是  $(0, 0)$ ，台风是  $(4, -2)$ 。假设台风向西（左）移动时，概率为 0.1，和向北（上）移动时，概率为 0.9，而且只能在移动一个单位距离后才可改变方向，请问这个台风遇到香港的概率是多少？（答案需准确至四位有效数字。）

**Hong Kong Mathematics Olympiad (2022/23)**  
**Finals (Group – Event 3)**

---

**FOR OFFICIAL USE**

Score for accuracy	<div style="border: 1px solid black; width: 40px; height: 30px; margin: 0 auto;"></div>	×	Mult. factor for speed	<div style="border: 1px solid black; width: 40px; height: 30px; margin: 0 auto;"></div>	=	<div style="border: 1px solid black; width: 40px; height: 30px; margin: 0 auto;"></div>		Team No.	<div style="border: 1px solid black; width: 40px; height: 30px; margin: 0 auto;"></div>	
				+		Bonus Score	<div style="border: 1px solid black; width: 40px; height: 30px; margin: 0 auto;"></div>	Time	<div style="border: 1px solid black; width: 40px; height: 30px; margin: 0 auto;"></div>	<div style="border: 1px solid black; width: 40px; height: 30px; margin: 0 auto;"></div>
						Total Score	<div style="border: 1px solid black; width: 40px; height: 30px; margin: 0 auto;"></div>		Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特别声明，答案须用数字表达，并化至最简。

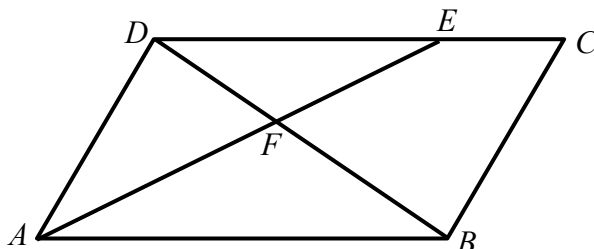
1. Let  $a_n$  be a sequence such that  $a_n = \frac{1}{(n+1)\sqrt{n} + n\sqrt{n+1}}$ . Find the value of  $s$  where  $s = a_1 + a_2 + a_3 + \cdots + a_{120}$ .

设  $a_n$  为序列且  $a_n = \frac{1}{(n+1)\sqrt{n} + n\sqrt{n+1}}$ 。如果  $s = a_1 + a_2 + a_3 + \cdots + a_{120}$ ，求  $s$  的值。

$s =$

2. Let  $ABCD$  be a parallelogram with  $AB = 40$ ,  $AD = 24$  and  $DB = 56$ . The angle bisector of  $\angle DAB$  meets side  $DC$  at the point  $E$ , and the diagonal  $DB$  meets  $AE$  at the point  $F$ . If the area of  $ABCD$  is 560 square units, find the area of the quadrilateral  $ECBF$ .

设  $ABCD$  为平行四边形且  $AB = 40$ ,  $AD = 24$  及  $DB = 56$ 。 $\angle DAB$  的角平分线与  $DC$  相交于  $E$  点，且对角线  $DB$  与  $AE$  相交于  $F$  点。求四边形  $ECBF$  的面积。



3. Let  $f(x)$  be a function such that

$$f(x) + f\left(-\frac{1}{x-1}\right) = \frac{2x}{3} + \frac{5}{3} + f\left(1 - \frac{1}{x}\right), \quad x \neq 0, 1$$

Find the value of  $f(-1)$ .

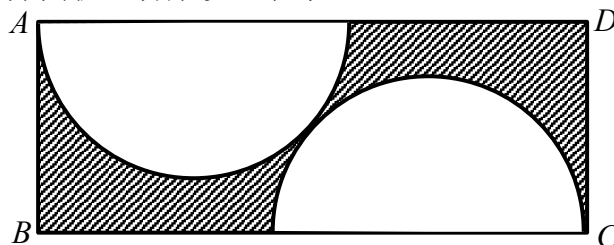
设  $f(x)$  为函数并满足

$$f(x) + f\left(-\frac{1}{x-1}\right) = \frac{2x}{3} + \frac{5}{3} + f\left(1 - \frac{1}{x}\right), \quad x \neq 0, 1$$

求  $f(-1)$  的值。

4. In the following figure,  $ABCD$  is a rectangle. The two semi-circles are identical and they are tangent to each other. If  $AB = 2$  and  $BC = 6$ , find the area of the shaded part in terms of  $\pi$ .

下图中， $ABCD$  是一个长方形。两个半圆形完全相等且它们彼此相切。如果  $AB = 2$  及  $BC = 6$ ，求图中阴影部分面积（答案以  $\pi$  表示）。



**Hong Kong Mathematics Olympiad (2022/23)**  
**Finals (Group – Event 4)**

---

**FOR OFFICIAL USE**

Score for accuracy	<div style="border: 1px solid black; width: 40px; height: 30px; display: inline-block;"></div>	×	Mult. factor for speed	<div style="border: 1px solid black; width: 40px; height: 30px; display: inline-block;"></div>	=	<div style="border: 1px solid black; width: 40px; height: 30px; display: inline-block;"></div>		Team No.	<div style="border: 1px solid black; width: 40px; height: 30px; display: inline-block;"></div>	
				+		Bonus Score	<div style="border: 1px solid black; width: 40px; height: 30px; display: inline-block;"></div>	Time	<div style="border: 1px solid black; width: 40px; height: 30px; display: inline-block;"></div>	<div style="border: 1px solid black; width: 40px; height: 30px; display: inline-block;"></div>
						Total Score	<div style="border: 1px solid black; width: 40px; height: 30px; display: inline-block;"></div>	Min.	Sec.	



Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特别声明，答案须用数字表达，并化至最简。

1. Find the product of all the real roots of the equation  $x^{\log_{10} x} = 10$ .

求方程  $x^{\log_{10} x} = 10$  所有实根的积。

2. Let  $p$  be a prime and  $m$  be an integer. If  $p(p+m)+2p=(m+2)^3$ , find the greatest possible value of  $m$ .

设  $p$  为质数及  $m$  为整数。如果  $p(p+m)+2p=(m+2)^3$ ，求  $m$  的最大值。

3. If the length of one side of a regular tetrahedron is 1, find the volume of such tetrahedron.

如果正四面体的边长是 1，求该正四面体的体积。

4. Let  $P$  be the product of 3659893456789325678 and 342973489379256. Find the number of digits of  $P$ .

设  $P$  为 3659893456789325678 和 342973489379256 的乘积。求  $P$  的位数。